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On the topological properties of the world trade web: A weighted network analysis[☆]

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Abstract

This paper studies the topological properties of the World Trade Web (WTW) and its evolution over time by employing a weighted-network analysis. We show that the WTW, viewed as a weighted network, displays statistical features that are very different from those obtained by using a traditional binary-network approach. In particular, we find that: (i) the majority of existing links are associated to weak trade relationships; (ii) the weighted WTW is only weakly disassortative; (iii) countries holding more intense trade relationships are more clustered.

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1. Introduction

In the past years, a large number of contributions have empirically explored the topological properties of many real-world networks [1–4]. Within this exploding body of literature, econophysicists have devoted considerable attention to the World Trade Web (WTW), see Refs. [5–9]. In these studies, the WTW is defined as the network of world-trade relations where countries play the role of nodes and a link between any two countries is in place if and only if there exists a non-zero – or a sufficiently intense[1] – import/export flow between them in a given year. The picture stemming from these empirical investigations can be summarized as follows [5–7]. First, the WTW is characterized by a disassortative pattern: countries with many trade partners are on average connected with countries holding

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1 For example, in [9] it is assumed that a link exists between countries i and j if the value of exports from country i to country j (as a proportion of country i's total exports) is greater than a given percentage.

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few partners. Second, partners of well-connected countries are less interconnected (among themselves) than those of poorly-connected ones, implying some hierarchical arrangements. Third, the structural properties of the WTW have remained remarkably stable over time. In terms of network statistics, two main stylized facts seem therefore to robustly emerge across the years: (SF1) node degree (see Ref. [1], p. 49) and average nearest-neighbor degree [10] are negatively correlated; (SF2) node degree and clustering coefficient [11] are negatively correlated.

To a large extent, however, SF1-2 refer to a binary-network analysis (BNA).[2] Indeed, Refs. [5–7] only study the WTW as a network where each link from country i to country j either exists or not. A binary network can thus be characterized by a binary adjacency matrix A, whose generic entry $a_{ij} = 1$ if and only if a link from node i to j is in place.[3] A BNA treats all links in the WTW as if they were completely homogeneous. This is counterintuitive, as actual import–export flows greatly differ both when they are evaluated in their levels (e.g. in current US dollars) and when they are computed as shares of the importing/exporting country size (measured e.g. by its gross domestic product, GDP). In order to take into account the existing heterogeneity in the capacity and intensity of connections, a weighted-network analysis (WNA) can instead be performed.[4] More formally, in a WNA each existing link is assigned a value $w_{ij} > 0$ proportional to the weight of that link. Hence, a *weighted* network is fully described by its $N \times N$ weight matrix $W = \{w_{ij}\}$, where $w_{ii} = 0$ for all i.

In this paper, we explore the statistical properties of the WTW using a WNA. We ask whether the two stylized facts above still hold when one weights each existing link with some proxy of the actual trade flow flowing through it. Our results show that SF1-2 are not robust to a WNA. In particular, the WTW viewed as a weighted network is only weakly disassortative. Furthermore, better-connected countries tend to be *more* clustered. The only statistical feature which resists in a WNA is the constancy over time of WTW properties. The rest of the paper is organized as follows. In Section 2 we describe the data and we define network statistics. Section 3 discusses our main results. Finally, Section 4 concludes the paper.

2. Data and network statistics

We employ international trade data provided by [17] to build a time-sequence of weighted directed networks (see Ref. [18], for details). Our sample refers to $T = 20$ years (1981–2000) and $N = 159$ countries. For each country and year, data report trade flows in current US dollars. To build adjacency and weight matrices, we followed the flow of goods. This means that rows represent exporting countries, whereas columns stand for importing countries. We define a "trade relationship" by setting the generic entry of the adjacency (binary) matrix $\tilde{a}^t_{ij} = 1$ if and only if exports from country i to country j (labeled by e^t_{ij}) are strictly positive in year t. Link weights are instead defined as $\tilde{w}^t_{ij} = e^t_{ij}/\mathrm{GDP}^t_i$, i.e. exports over GDP of the exporting country.[5] For any particular choice of the weighting setup, we end up with a sequence of $N \times N$ adjacency and weight matrices $\{\tilde{A}^t, \tilde{W}^t\}$, $t = 1981, \ldots, 2000$, which fully describe the evolution of the WTW from a binary and weighted directed perspective.

A preliminary statistical analysis of both binary and weighted matrices suggests that $(\tilde{A}^t, \tilde{W}^t)$ are sufficiently symmetric to justify an undirected analysis (see Refs. [18,19], for details). Therefore, we define entries of the symmetrized adjacency matrix $a_{ij} = 1$ if and only if either $\tilde{a}_{ij} = 1$ or $\tilde{a}_{ji} = 1$ (and zero otherwise). Accordingly, the generic entry of the symmetrized weight matrix W is defined as $w^t_{ij} = \frac{1}{2}(\tilde{w}^t_{ij} + \tilde{w}^t_{ji})$. Finally, in order to have $w^t_{ij} \in [0, 1]$ for all (i, j) and t, we renormalize all entries in W by their maximum value.

In this paper, we present results concerning three network statistics [1,10,11,15,20,21]. First, for any node i, we compute its *node degree*, defined as $\mathrm{ND}_i = A_{(i)}\mathbf{1}$, where $A_{(i)}$ is the ith row of A and $\mathbf{1}$ is a unary vector. Node degree can be naturally extended to weighted networks by computing node strength $\mathrm{NS}_i = W_{(i)}\mathbf{1}$, where again $W_{(i)}$ is the ith row of W. While ND tells us how many partners a node holds, NS gives us an idea of how intense these relationships are. Second, we define *average nearest-neighbor degree* of a node as $\mathrm{ANND}_i = (A_{(i)}A\mathbf{1})/(A_{(i)}\mathbf{1})$. In the case of weighted networks, this indicator becomes the average nearest-neighbor strength and is defined as

[2] See Ref. [8] for an exception.

[3] Self-loops, i.e. links connecting i with itself are not typically considered. This means that $a_{ii} = 0$, for all i.

[4] See among others Refs. [12–16]. Real-world networks that have been studied via a WNA include the worldwide airport network, scientific collaborations and social acquaintances.

[5] All our results are robust to alternative weighting schemes. For example, we experimented with weights defined as $e^t_{ij}/\mathrm{GDP}^t_j$, or simply as e^t_{ij}.

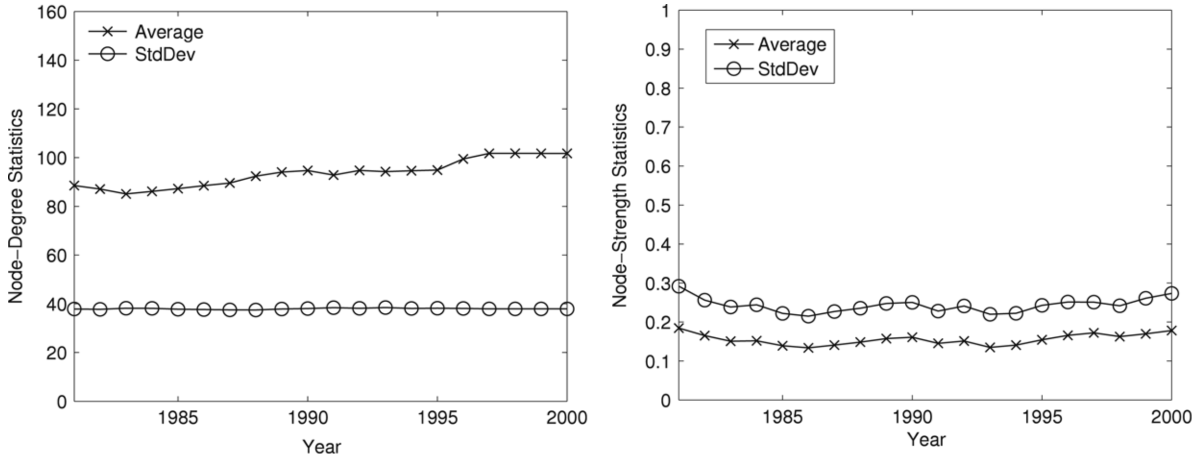

Fig. 1. Average and standard deviation of average node degree (left) and node strength (right) vs. years.

$\text{ANNS}_i = (A_{(i)}W\mathbf{1})/(A_{(i)}\mathbf{1})$. ANND measures the average number of partners of a given node's partners, while ANNS tells us how intense are the relationships maintained by the partners of a given node. Third, we employ the *clustering coefficient*, defined for binary networks as $\text{BCC}_i = (A^3)_{ii}/(\text{ND}_i(\text{ND}_i - 1))$ and for weighted networks as $\text{WCC}_i = (W^{\left[\frac{1}{3}\right]})^3_{ii}/(\text{ND}_i(\text{ND}_i - 1))$. Here $(A^3)_{ii}$ is the ith entry on the main diagonal of $A \cdot A \cdot A$ and $W^{\left[\frac{1}{3}\right]}$ stands for the matrix obtained from W after raising each entry to $1/3$ (see Ref. [21], for a discussion). BCC counts the fraction of a node's partners that are themselves partners, while WCC measures how much intense are the interactions among three strongly-connected partners.

3. Results

We begin by investigating the behavior of ND and NS distributions. As Fig. 1 shows, the WTW as a binary network is very densely connected. On average each country holds about 90 trade partners (over a maximum of 159). Conversely, the weighted WTW displays, on a [0, 1] scale, a relatively low average NS. Notice also that the first two moments of both ND and NS have remained relatively stable over time (if any, average ND has been slowly increasing). This is a general finding: the first four moments of all three indicators discussed in Section 2, and their correlations, display a marked time stationarity. This implies that the structural properties of the WTW, viewed either as a binary or as a weighted network, have not been influenced by the process of globalization (however this may be defined), and confirms the results in Ref. [7]. Given this time stationarity, in the rest of the paper we will focus on a representative year (2000).

The fact that average ND is relatively high, whilst average NS is low, suggests that the majority of existing connections are relatively weak. Indeed, the ND–NS correlation coefficient for the period 1981–2000 is on average 0.50. The structural difference between ND and NS can be better grasped by plotting the kernel-smoothed ND and NS density, see Fig. 2 for year 2000. The ND distribution is relatively left-skewed, with a modal value around 90. However, there exists a group of countries that trade with almost everyone else in the sample (hence, the second peak around 150).

This picture changes substantially in the weighted case. The NS distribution is in fact left-skewed: many weak trade relationships coexist with a few strong ones. Size-rank plots show that NS distributions are in fact log-normal in the body and Pareto in the upper tail. This result indicates the presence of a relevant heterogeneity in the intensity of trade interactions and suggests that a WNA can provide a more complete description of the topological properties of the WTW with respect to a BNA.

Let us now turn to SF1. As shown in Refs. [5,7], the WTW seems to display a disassortative pattern: countries with many trade partners are on average connected with countries holding few partners. This is confirmed in our data. The ND–ANND correlation coefficient is on average -0.95 across the years. A scatter plot for year 2000 shows how strong the disassortative pattern is, see Fig. 3, left panel. However, when we plot for the same year ANNS vs. NS, the correlation pattern appears to be substantially weaker, cf. Fig. 3, right panel. Countries with medium-low NS are in fact

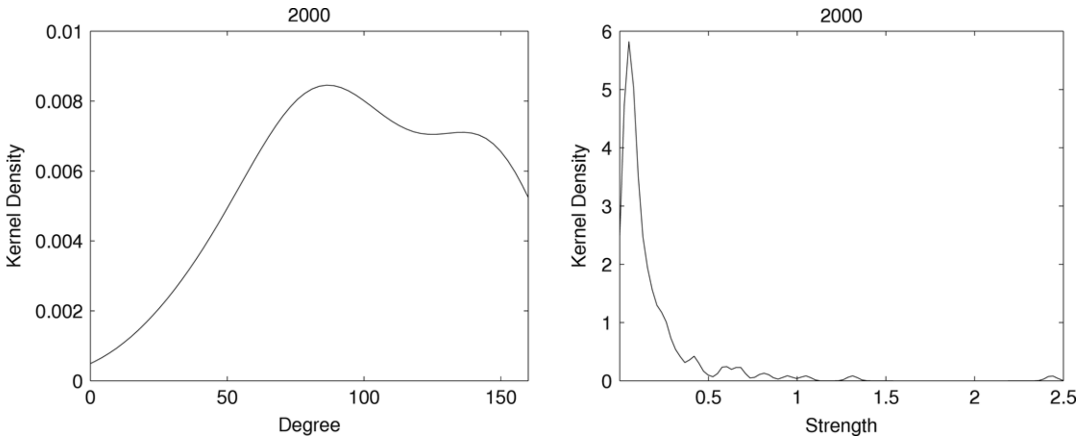

Fig. 2. Kernel-smoothed density of node degree (left) and node strength (right) distributions. Year: 2000.

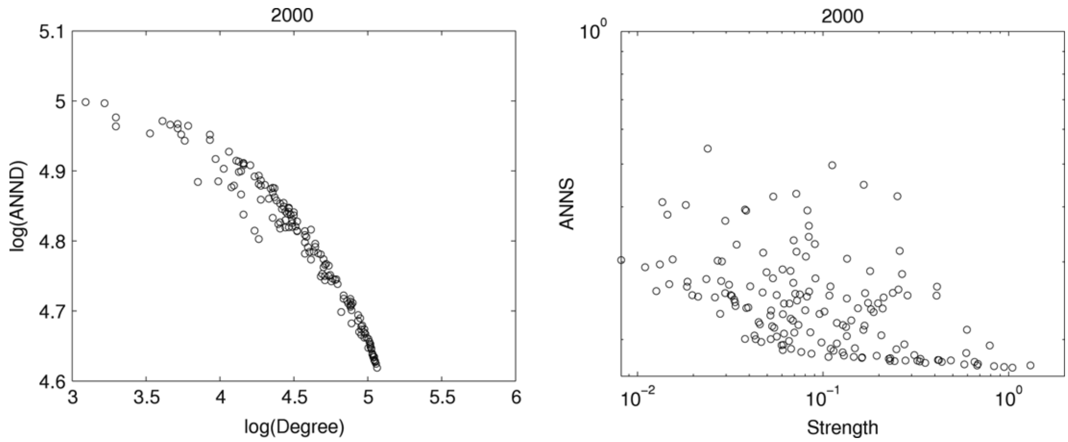

Fig. 3. Assortativity analysis. Left: ANND vs. node degree (in log). Right: ANNS vs. node strength (axes in log scale). Year: 2000.

characterized by a wide range of ANNS values, meaning that there can be well-connected countries that trade with partners that are also well-connected. Indeed, the ANNS–NS correlation coefficient stays around −0.40 for the whole period. Therefore, SF1 does not seem to hold that robustly when the WTW is studied from a weighted-network perspective.

Finally, we address the issue whether SF2 is confirmed by a WNA. According to Refs. [7,5], countries that in a binary WTW hold more trade partners are typically associated to lower clustering coefficients. This means that any two partners (h, k) of a given country i are not very likely to establish a trade relationship (i.e., a_{hk} is likely to be zero). Again, a BNA confirms these results for our data. From a binary perspective, there exists a −0.96 correlation (stable over time) between BCC and ND (Fig. 4, top-left panel). In fact, the average BCC is very high (about 0.8 on a [0, 1] scale). A scatter plot of BCC vs. ND in year 2000 further corroborates SF2 (Fig. 4, bottom-left panel).

Nevertheless, a WNA gives here the opposite result. If viewed as a weighted network, the WTW displays an increasing, positive, and significant correlation between WCC and NS (Fig. 4, top-right panel), which results in upward-sloping WCC–NS scatter plots (Fig. 4, bottom-right panel). Average WCC is actually very low (about 1E-03 on the same [0, 1] scale). Therefore, once the existing heterogeneity of trade relationships is taken into account through a weighted approach, one finds that countries holding more intense relationships are *more likely* to form strongly-connected trade triangles. In other words, trade clubs (or cliques) are typically the case in the WTW.

4. Concluding remarks

In this paper we have performed a WNA to investigate the topological properties of the WTW in the period 1981–2000. We have studied network connectivity, assortativity and clustering, and we have compared our weighted-

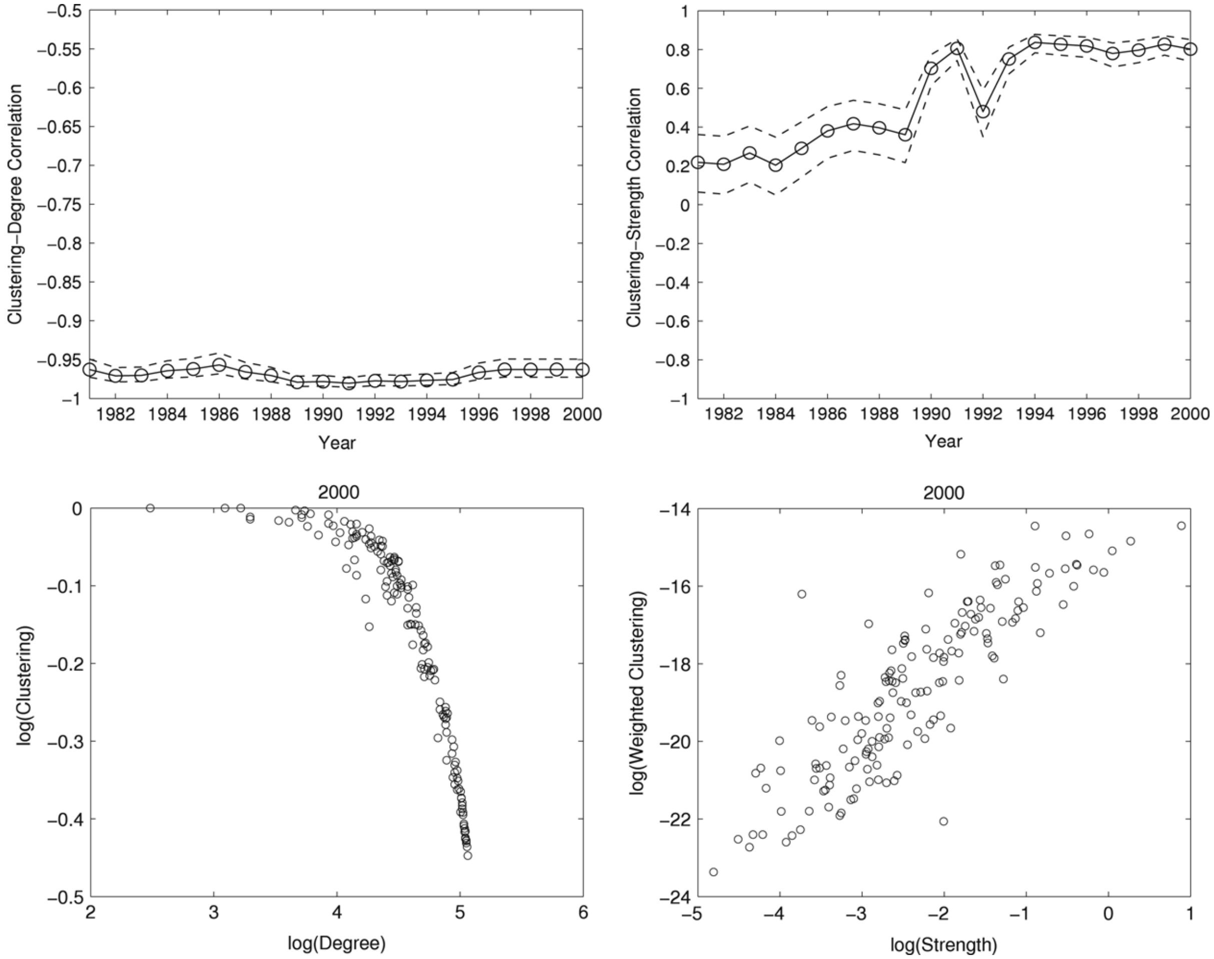

Fig. 4. Clustering analysis. Top-Left: Correlation between BCC and node degree over time. Top-Right: Correlation between WCC and node strength over time. Bottom-Left: BCC–ND (log) scatter plot in year 2000. Bottom-Right: WCC–NS (log) scatter plot in year 2000. Dashed lines in top panels: 5% and 95% confidence intervals.

Table 1
Main results

	Connectivity	Assortativity	Clustering
BNA	Highly-connected + Bimodal ND distributions	Strongly disassortative	Highly-clustered + Negative CC–ND correlation
WNA	Weakly-connected + Skewed ND distributions	Weakly disassortative	Weakly-clustered + Positive CC–ND correlation

network results to those obtained using a BNA. We have shown that the picture stemming from a WNA is substantially different from that obtained using a BNA. Our results are summarized in Table 1.

The above findings support the idea that accounting for the heterogeneity of interaction intensity in networks is crucial to better understand their complex architecture [13,14]. In our application to the WTW, for example, the binary representation of the WTW leads to a highly-connected graph, where all links have the same impact on the resulting statistics. Almost by definition, this implies very large values of ANND and clustering coefficients for the majority of nodes. Therefore, the computation of correlation patterns is somewhat biased by this high-connectivity level. However, different links carry a very different interaction intensity in the WTW: the majority of links are indeed associated to low import/export flows (e.g., as a percentage of exporter GDP). This is true irrespective of the particular method one may use to weight the links. By exploiting this additional information, a WNA allows one to better grasp the underlying topological structure of the network under study.

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